

3. Dynamics

The vertically-integrated, time-mean momentum equations can be written:

$$A^x - fV = -P_x + \tau^x + F^x \quad (1a)$$

$$A^y + fU = -P_y + \tau^y + F^y \quad (1b)$$

where upper case symbols indicate vertically-integrated quantities. (U, V) are the velocity components, P is the pressure and τ is the wind stress. $A = (A^x, A^y) = \int \nabla \bullet \mathbf{u} \mathbf{u} \, dz$ are the advective terms and $F = (F^x, F^y)$ are the (combined) friction terms. Time means must be taken after forming products such as the definition of A . Subscripts indicate partial differentiation, and factors of ρ_0 have been dropped for simplicity of notation. The vertically-integrated mean continuity equation is:

$$U_x + V_y = 0 \quad (2)$$

A common linearization of (1) retains only the Coriolis, pressure gradient and wind stress terms:

$$-fV = -P_x + \tau^x \quad (3a)$$

$$fU = -P_y + \tau^y \quad (3b)$$

Taking the curl of (3), substituting from (2) gives the meridional transport associated with the Sverdrup balance:

$$\beta V = \text{Curl}(\tau) \quad (4)$$

The Sverdrup zonal transport is found from (2), using (4) for V and integrating from the eastern boundary where U is zero:

$$U = \frac{1}{\beta} \int_{EB}^x \text{Curl}(\tau)_y \, dx + U_0(y) \quad (5)$$

where $U_0(y)$ is the eastern boundary condition (see Appendix). The success of the Sverdrup balance in explaining major features of the ocean circulation points to the fundamental importance of the vorticity in the large-scale dynamics.

In section 4 we will be interested in diagnosing the importance of the advective and friction terms in the ocean GCM (section 2c). A simple approach might be to compare the

terms of (1), noting the size of the advective and friction terms compared to the leading Coriolis, pressure gradient and wind stress terms. It will be shown that the second-order terms are small in the model solution (which gives a largely realistic picture of the observed currents), and that the vertically-integrated momentum balance is principally among the Ekman and geostrophic terms. However, at the same time the model currents will be seen to differ substantially from the Sverdrup transports. Resolving this apparent paradox is achieved by considering the effects of the second-order terms on the vorticity balance.

One way this can be accomplished is to treat the advective and friction terms as *forcing* terms; that is, to define a generalized stress $\tau^* \equiv \tau + \tau' + \tau''$, where $\tau' \equiv -\mathbf{A}$ and $\tau'' \equiv \mathbf{F}$. Equations (1) then are rewritten

$$-fV = -P_x + \tau^{*x} \quad (6a)$$

$$fU = -P_y + \tau^{*y} \quad (6b)$$

which have the same form as the linearized set (3). Taking the curl of (6) leads to a Sverdrup-like balance of the form (4) and (5), with τ replaced by τ^* , in which the effects of the advective and friction terms are evaluated through their modification of the vorticity. This procedure allows all three “forcing” terms to be studied individually and compared, and their effects linearly added to produce a complete solution. Of course, these are just manipulations of (1), so the Sverdrup-like U and V obtained from (6) recovers the original U and V . This is not a method to get solutions to (1), but simply to diagnose the importance of the terms in the context of the vorticity balance. What is found by this formulation is that the importance of the second-order terms comes through their derivatives; in particular, their effect on the zonal current is realized in $d[\text{Curl}(\tau^*)]/dy$, and these derivatives have quite different spatial patterns than the terms themselves. Although it will be seen that in the model solution the advective terms A^x and A^y in (1) have similar magnitudes, A^x has much greater significance because in forming the curl its y -derivatives are much larger than the x -derivatives of A^y .

APPENDIX

The boundary condition $U_\theta(y)$ in the integral (5) for the Sverdrup zonal transport is often assumed to be zero, which is only true for a meridionally-oriented eastern boundary. When $\text{Curl}(\tau)$ is non-zero at a tilted boundary, the Sverdrup relation (4) would imply flow

normal to the boundary unless there was a corresponding zonal transport U_0 to make the total boundary flow exactly alongshore. This required value of U along the coast is the boundary condition for the integral (5).

This boundary condition can be found using the Sverdrup streamfunction

$$\psi = \frac{1}{\beta} \int_{x_e(y)}^x \text{Curl}(\tau) dx, \quad V = \psi_x, \quad U = -\psi_y \quad (\text{A1})$$

where $x_e(y)$ is the longitude of the boundary at each latitude. The boundary condition for (A1) is $\psi = \text{constant}$, no matter what the boundary slope, since the no-normal flow condition precludes any ψ contours from intersecting the coast, and we choose $\psi = 0$ along the American coast. The meridional derivative of (A1) gives the complete expression for U (using Liebniz' Rule):

$$U = -\psi_y = -\frac{1}{\beta} \left(\int_{x_e(y)}^x \text{Curl}(\tau)_y dx - d[x_e(y)]/dy \text{Curl}(\tau)|_{x=x_e(y)} \right) \quad (\text{A2})$$

where the first term on the right hand side is the contribution to U from interior wind forcing, and the second term is the value of U on the boundary (U_0 in equation (5)). $d[x_e(y)]/dy$ in that term is the boundary slope, which is zero for a meridional coast and positive clockwise.

On a meridionally-oriented coast, the boundary condition is $U_0 = 0$, but in general the value is non-zero. For the ERS scatterometer winds and the shape of the American coast, the values of U_0 are found to be small compared to the interior term, except along the coast of Central America at 8° to 10°N where $\text{Curl}(\tau)$ is large and positive (associated with the Papagayo winds; see Kessler (2001)) and the coast is strongly tilted to the west. In this region the values of U_0 derived from (A2) were about $-10 \text{ m}^2 \text{ s}^{-1}$, which is generally smaller than, but comparable to, the interior signal. It is also noted that the above formulation is inadequate to encompass a zonally-oriented coast, where the slope term becomes infinite. In that case the dynamics expressed in (2) and (3) that lead to the Sverdrup balance have no steady solution, which would require additional terms, such as friction. This is relevant to the tropical Pacific since the coastline of Mexico is zonal at the Gulf of Tehuantepec (near 16°N), which is also a region where the mean wind stress curl is large.